Towards Compressive Sensing for Ground-to-Air Monostatic Radar

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***Abstract*—In principle, the problem of range-velocity estimation of moving targets in radar may be solved efficiently by compressed sensing (CS) under favorable conditions. Unfortunately, the performance of CS radar degrades severely in realistic scenarios where the signal-to-noise-ratio (SNR) is low. In this paper, we investigate how coherent integration can be utilized to improve the performance of CS at low SNR. Numerical results indicate that coherent integration boosts the effective SNR, leading to reliable CS reconstructions even at low SNR.**

I. Introduction

A basic application of radar is the estimation of *range* and *velocity* of moving targets [1]. The target space is often models as a time-varying linear system $\mathbf{H}$, which is estimated from the reflection $\mathbf{y}$ of the probing signal $\mathbf{f}$. From the reflected signal $\mathbf{y} = \mathbf{Hf}$, the time (range) and Doppler (velocity) shifts of the targets are inferred.

In case of airborne targets, there will be very few targets against a wide sky. Usually the number of targets in the search range of a radar is unknown. Traditionally, matched filter is used to construct the target space whose resolution is limited by the time-frequency uncertainty of the probing signal [2]. Compressed Sensing (CS) solves this problem from an entirely different mathematical standpoint, where *sparsity* of the target scene plays a key role. CS radar provides a higher resolution of the target space under certain favorable conditions based on the sparsity and incoherence of the probing signal [3]. These conditions are often violated in realistic scenarios due to the presence of noise and clutter, and hence CS radar may produce output that deviates significantly from the ground truth. Most studies on CS Radar [3]–[5] consider unrealistic conditions with high signal-to-noise-ratio (SNR). It is important to investigate the CS radar framework in a more realistic regime for accurate target detection. In this paper, we study the effect of noise on CS radar, and investigate how coherent integration can be combined with CS radar to overcome noise.

II. Matched filtering and CS approach

In matched filtering, the radar return is correlated with a time-frequency shifted version of the probing signal, resulting in a time-frequency (range-Doppler) plane. In the case of multiple targets, the range-Doppler plane consists of the superposition of self-ambiguity functions of the probing signal centered at the target locations. The spread of the self-ambiguity function blurs the target location and adds uncertainty to the number of targets in the range-Doppler plane, as indicated by the time-frequency uncertainty principle.

In compressed sensing radar, the target scene $\mathbf{H}$ is assumed to be sparse, and is the solution of an underdetermined linear inverse problem described by a matrix $\mathbf{\Phi}$. That inverse problem can be solved by convex optimization, and perfect recovery is possible under two conditions: (1) the target scene $\mathbf{H}$ is sufficiently sparse; (2) the matrix $\mathbf{\Phi}$ is incoherent. The first condition is usually fulfilled, as the number of targets is often relatively small. The second condition is determined by the probing waveform and the basis chosen for representation. In case of noisy observations $\mathbf{y}$, CS radar can reconstruct up to a tolerance level depending on the SNR.

III. CS Radar and Coherent Integration

Usually, single-pulse radar returns have very low SNRs, and in such cases CS fails miserably [6]. Clearly, the SNR needs to be boosted before applying CS.

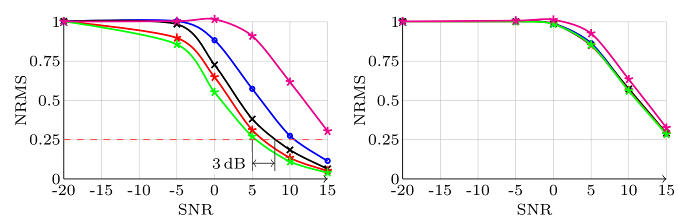
So far, CS radar has been considered separately, as a replacement for standard approaches. In contrast, we view CS as a scheme that can coexist with the current techniques, supplement them, and improve the accuracy of the overall radar system.

As a first step in that direction, here we blend coherent integration with CS reconstruction in two different settings. Firs, the N single pulse radar returns are integrated before applying CS reconstruction, referred as *Integration before CS*. Second, we apply the CS reconstruction to single pulse radar returns and integrate the range-Doppler plane for N successive pulses, referred as *Integration after CS*.

IV. Results and Discussion

The preliminary results are displayed in Fig. 1. If coherent integration is carried out before CS, the effective SNR is increased by $10\log_{10}(N)$ and the reconstruction error drops accordingly. On the other hand, coherent integration of the CS reconstructions from single returns does not significantly reduce the error.

A promising next step would be to merge matched filtering with CS reconstruction, where the output of the matched filter is used to guide the CS reconstruction algorithms (as a Bayesian prior [7]).



(a) Integration before CS (b) Integration after CS

Fig. 1. Reconstruction error performance for integration (a) before, and (b) after CS reconstruction for the different coherent integration durations, N = 1 (magenta), N = 5 (blue), N = 10 (black), N = 15 (red), and N = 20 (green). NRMS stands for normalized root-mean-square between the original and reconstructed target scene.

References

[1] M. Skolnik, *Introduction To Radar Systems 3/E*. Tata McGraw Hill, 2003.

[2] M. A. Richards, *Fundamentals Of Radar Signal Processing*. McGraw-Hill Education (India) Pvt. Limited, 2005.

[3] M. A. Herman and T. Strohmer, "High-resolution radar via compressed sensing," *IEEE Trans. on Signal Processing*, vol. 57, no. 6, pp. 2275–2284, Jun. 2009.

[4] W. U. Bajwa, K. Gedalyahu, and Y. C. Eldar, "Identification of parametric underspread linear systems and super-resolution radar," *IEEE Trans. on Signal Processing*, vol. 59, no. 6, pp. 2548–2561, Jun. 2011.

[5] A. Fannjiang and H.-C. Tseng, "Compressive radar with off-grid and extended targets," *arXiv:[209.6399v]*, Sep. 2012.

[6] J. H. Ender, "On compressive sensing applied to radar," *Signal Processing*, vol. 90, pp. 1402 – 1414, 2010.

[7] S. Ji, Y. Xue, and L. Carin, "Bayesian compressive sensing," *IEEE Trans. on signal proc.*, vol. 56, no. 6, pp. 2346–2356, Jun. 2008.